HAUSDORFF SCHOOL
FOR ADVANCED STUDIES
IN MATHEMATICS

UNIVERSITÄT BONN

Hausdorff School

"Perverse Sheaves in Enumerative Geometry"

10 to 14 February 2020

organized by
Georg Oberdieck, Johannes Schmitt

Abstracts

Pierrick Bousseau (ETH Zürich)

Quasimodular forms from Betti numbers

Abstract: This talk will be about refined curve counting on local P^2, the noncompact Calabi-Yau 3-fold total space of the canonical line bundle of the projective plane. I will explain how to construct quasimodular forms starting from Betti numbers of moduli spaces of one-dimensional coherent sheaves on P^2. This gives a proof of some stringy predictions about the refined topological string theory of local P^2 in the Nekrasov-Shatashvili limit. Partly based on work in progress with Honglu Fan, Shuai Guo, and Longting Wu.

Ben Davison (University of Edinburgh)

An introduction to BPS Lie algebras

Abstract: I will define and explain the perverse filtration on the cohomological Hall algebra that appears in Sven Meinhardt's talks. This perverse filtration is a result of "hidden properness" of the stack of representations of a quiver, which I will also explain. Using this filtration I'll define the BPS Lie algebra, which turns out to be the first piece of the perverse filtration.

Positivity for DT invariants of finite-dimensional Jacobi algebras

Abstract: A conjecture of Brown and Wemyss states (roughly) that all finite-dimensional Jacobi algebras arise from flopping curve contractions, so that via a chain of equalities due to Katz, Toda and others, the DT invariants of these algebras genuinely count rational curves (without signs or virtual class trickery) in deformed threefolds. In particular, the Brown-Wemyss conjecture implies that the DT invariants of finite-dimensional Jacobi algebras are always positive. I'll explain how to prove this positivity conjecture by realising (refined) DT invariants as the hypercohomology of BPS sheaves, and using cohomological wall crossing - joint work with Sven Meinhardt.

Albrecht Klemm (University of Bonn)

Topological String on genus one fibred CY 3-folds with N-sections and Jacobi forms

Abstract: The all genus topological string amplitudes on Calabi-Yau 3-folds are generating functions of the symplectic invariants of holomorphic curves. We argue that the Fourier-Mukai transform on the A-model category as well as the holomorphic anomaly of the B-model restrict these amplitudes to be meromorphic Jacobi-forms of $\Gamma_1(N)$ subgroups of $\mathrm{SL}(2, Z)$. Vanishing conditions and in the non-compact case Nakajima's blow up equations allow to fix the amplitudes explicetly.

Sven Meinhardt (University of Sheffield)

Minicourse: Vanishing cycles

Abstract:
1. The vanishing cycle sheaf: I will give a gentle introduction to vanishing cycle sheaves with as much intuition as possible. Basic canonical morphisms for algebraic maps and tensor products will be given and the Thom-Sebastiani theorem will be stated.
2. The vanishing cycle functor: I will sketch how to extend the vanishing cycle sheaf to a functor and discuss the usual behaviour under smooth or proper morphisms. A generalized Thom-Sebastiani theorem will be stated.
3. Vanishing cycle sheaves and Cohomological Hall algebras: I recall the concept of (weak) monoidal functors and show how vanishing cycle sheaves can be used to construct such functors using results from the 1st talk. With such a functor at hand, I introduce the Cohomological Hall algebra on the coarse moduli space of representations of quiver with potential or of vector bundles on curves.

Orientation data and Cohomological Hall algebras

Abstract: I discuss how to generalize the construction of monoidal functors given in the 3rd talk to moduli spaces of coherent sheaves etc. by gluing local vanishing cycle sheaves together. This requires the concept of orientation data which will be introduced.

Luca Migliorini (University of Bologna)

Minicourse: Introduction to perverse sheaves

Abstract: Given an algebraic variety X over a field, I will show how the category of perverse sheaves on X arises naturally by looking for a subcategory of the derived category of complexes of sheaves on X (with constructible cohomology sheaves), which is stable under Verdier duality and whose objects satisfy the weak Lefschetz property, namely the cohomology of their restriction to affine subsets vanishes when the degree exceeds the dimension. I will then try to give an idea why this subcategory is abelian artinian and noetherian, and describe its simple objects, the intersection cohomology complexes.

Supports of a map and higher discriminants

Abstract: After recalling the statement of the decomposition theorem of Beilinson Bernstein Deligne and Gabber, I will define the supports of a map in the sense of Ngô, then show how the notion of higher discriminants of a map, introduced by V. Shende and myself, is a useful tool to find this supports.

Junliang Shen (MIT)

An example of the Goparkumar-Vafa theory: the KKV formula revisited

Abstract: Curve counting invariants for K3 surfaces have been studied intensively over decades. The Katz-Klemm-Vafa (KKV) formula, proven by Pandharipande and Thomas in 2014, implies that these invariants satisfy a mysterious multiple cover structure and the modularities. In this talk, I will first recall a special case of the Gopakumar-Vafa proposal for curve counting invariants when the moduli space is smooth. Then as an example, I will present a proof of the KKV formula via the GV proposal using perverse sheaves. Based on joint work with Qizheng Yin.

Hitchin systems, hyper-Kaehler geometry, and the P=W conjecture

Abstract: Lagrangian fibrations play a central role in the study of hyper-Kaehler geometry, integrable systems, and curve counting invariants of local geometries. The P=W conjecture by de Cataldo, Hausel, and Migliorini suggests a surprising connection between the topology of Lagrangian fibrations and Hodge theory. This connection also implies the Gopakumar-Vafa conjecture for local curves. In this talk, we will first review a compact version of this phenomenon (related to my complementary talk). Then we will focus on interactions between compact and noncompact hyper-Kaehler geometry. Such connections , as well as ideas from multiple cover structures in curve counting theory of K3 surfaces, lead to new progress on the P=W conjecture for Hitchin systems and character varieties. This is joint work with Mark de Cataldo and Davesh Maulik.

Yukinobu Toda (IPMU, Tokyo)

Minicourse: Gopakumar-Vafa invariants via vanishing cycles

Abstract: In 1998, Gopakumar-Vafa predicted the existence of certain integer valued invariants on Calabi-Yau 3-folds (called GV invariants), which determine the Gromov-Witten series based on the string duality between type IIA and M theory. In this mini-course, I will talk about our approach (with D. Maulik) toward a mathematical definition of GV invariants via perverse sheaves of vanishing cycles, based on earlier ideas of Hosono-Saito-Takahashi and Kiem-Li. I will explain some necessary background for our definition, e.g. perverse sheaves of vanishing cycles, d-critical structures, and then give our definition of GV invariants. After this, I will explain several conjectures relating GV invariants and other curve counting invariants such as GW invariants, Pandharipande-Thomas invariants. A current status on these conjectures and several open problems will be also discussed.

Hall-type algebras for categorical Donaldson-Thomas theories on local surfaces

Abstract: I will introduce the notions of DT categories and PT categories for local surfaces, i.e. the total spaces of canonical line bundles on surfaces. Our constructions are regarded as gluing of categories of torus equivariant matrix factorizations via Koszul duality, and should recover the usual DT (PT) invariants by taking their periodic cyclic homologies. I will then show that the categorified cohomological Hall algebra structure on surfaces constructed by Porta-Sala descend to those on DT categories, giving a Hall-type algebra structure on DT categories. A similar argument also shows that PT categories admit actions of DT categories of zero dimensional sheaves. This action may yield a categorification of Wely algebra action of homologies of Hilbert schemes of points on locally planar curves due to Rennemo, which is relevant for GV formula of generating series of PT invariants.
